TUGAS KOLABORASI TIK & MATEMATIKA (APLIKASI INTEGRAL)

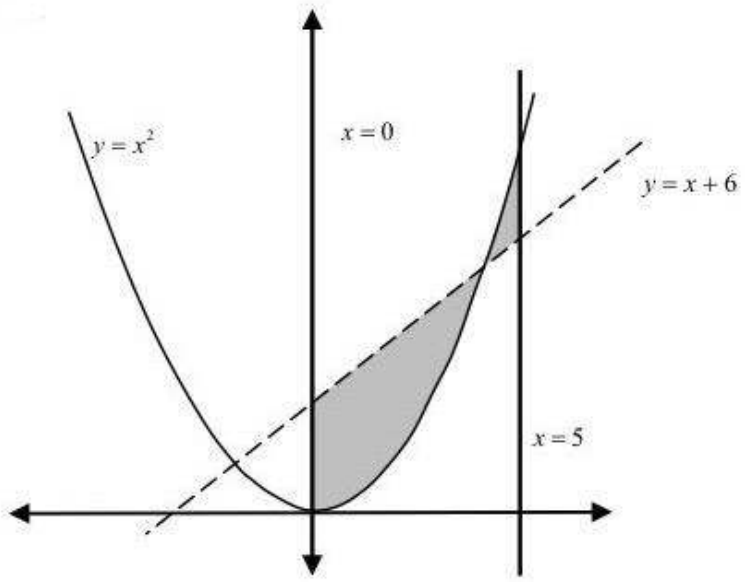
ANGGOTA :

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→ ***MEMBUAT 6 SOAL APLIKASI INTEGRAL***

A. 2 SOAL MENENTUKAN LUAS DENGAN DIKETAHUI GAMBAR DAERAH

1. *Perhatikan gambar berikut. Tentukan luas daerah yang diarsir.*



Cari titik potong:

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3 \text{ V } x = -2\ (TM)$

Cari luasan:

$$L = \int_0^3 [(x + 6) - (x^2)]\,dx + \int_3^5 [(x^2) - (x + 6)]\,dx$$

$$L = \int_0^3 (-x^2 + x + 6)\,dx + \int_3^5 (x^2 - x - 6)\,dx$$

$$L = \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x\right]_0^3 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x\right]_3^5$$

$$L = \left(-\frac{1}{3}(3)^3 + \frac{1}{2}(3^2) + 6(3)\right) - 0 + \left(\frac{1}{3}(5)^3 - \frac{1}{2}(5)^2 - 6(5)\right) -$$

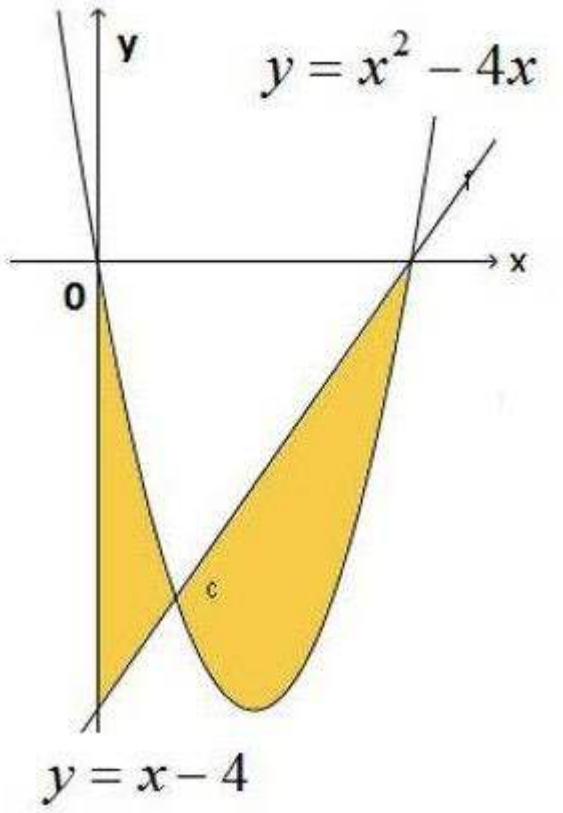
$$\left(\frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 - 6(3)\right)$$

$$L = (-9 + \frac{9}{2} + 18) + (\frac{125}{3} - \frac{25}{2} - 30) - (9 - \frac{9}{2} - 18)$$

$$L = \frac{27}{2} - \frac{5}{6} + \frac{27}{2}$$

$$\boxed{L = \frac{157}{6}\ satuan\ luas}$$

2. *Perhatikan gambar berikut. Tentukan luas daerah yang diarsir.*



Cari titik potong

$x - 4 = x^2 - 4x$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x = 4 \text{ V } x = 1$

* $x = 4,\ y = 0$

* $x = 1,\ y = -3$

$$L = \int_0^1 [(x^2 - 4x) - (x - 4)]\,dx + \int_1^4 [(x - 4) - (x^2 - 4x)]\,dx$$

$$L = \int_0^1 (x^2 - 5x + 4)\,dx + \int_1^4 (-x^2 + 5x - 4)\,dx$$

$$L = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\right]_0^1 + \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\right]_1^4$$

$$L = \left(\frac{1}{3}(1)^3 - \frac{5}{2}(1)^2 + 4(1)\right) - 0 + \left(-\frac{1}{3}(4^3) + \frac{5}{2}(4)^2 - 4(4)\right) - \left(-\frac{1}{3}(1)^3 + \frac{5}{2}(1)^2 - 4(1)\right)$$

$$L = \left(\frac{1}{3} - \frac{5}{2} + 4\right) + \left(-\frac{64}{3} + 40 - 16\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right)$$

$$L = \frac{11}{6} + \frac{8}{3} + \frac{11}{6}$$

$$\boxed{L = \frac{38}{6}\ satuan\ luas}$$

B. 2 SOAL MENENTUKAN LUAS TANPA GAMBAR

1. *Luas daerah yang dibatasi oleh kurva $y = x^2 - 2x + 1$ dan garis $y - x = 1$ adalah ...*

$y = x^2 - 2x + 1$

Cari titik puncak

$x = \frac{-b}{2a}$

$x = \frac{-(-2)}{2(1)}$

$x = 1$

$y = (1)^2 - 2 \cdot 1 + 1$

$y = 0$

$y - x = 1$

Cari titik puncak

y	0	1
x	3	4

$y = x^2 - 2x + 1$ $y - x = 1$

Titik puncak $(1, 0)$

Misal $y = 0$

$0 = x^2 - 2x + 1$

$(x - 1)(x - 1) = 0$

$x = 1$

$(1, 0)$

Cari titik potong

$y - x = 1$

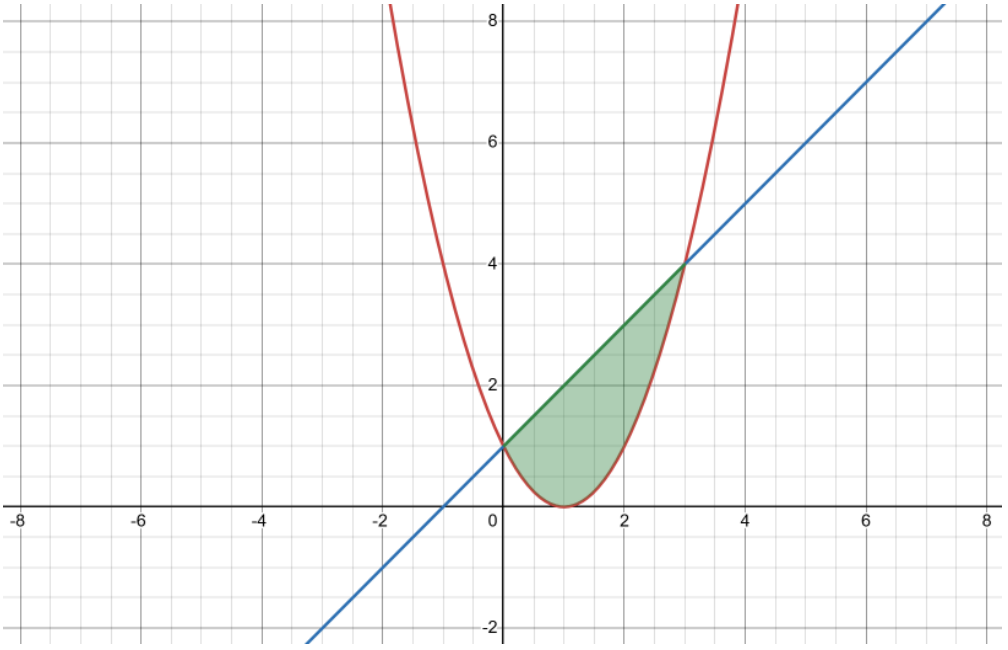
$y = 1 + x$

$x^2 - 2x + 1 = 1 + x$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0 \ v \ x = 3$



Cari luas

$$L = \int_0^3 [(x + 1) - (x^2 - 2x + 1)]\, dx$$

$$L = \int_0^3 [- x^2 + 3x]\, dx$$

$$L = [-\frac{1}{3}x^3 + \frac{3}{2}x^2]_0^3$$

$$L = [-\frac{1}{3}.3^3 + \frac{3}{2}.3^2] - 0$$

$$L = -(9) + \frac{27}{2}$$

$$\boxed{L = 4,5\ satuan\ luas}$$

2. *The area of the region bounded by the curve $y = x^2 - 6x + 5$ and $y = -x^2 + 5$ is ... square units.*

$y = x^2 - 6x + 5$

Cari titik puncak

$x = \frac{-b}{2a}$

$x = \frac{-(-6)}{2(1)}$

$x = 3$

$y = (3)^2 - 6 \cdot 3 + 5$

$y = -4$

Titik puncak $(3, -4)$

Misal $y = 0$

$y = -x^2 + 5$

Cari titik puncak

$x = \frac{-b}{2a}$

$x = \frac{-(0)}{2(-1)}$

$x = 0$

$y = -(0)^2 + 5$

$y = 5$

Titik puncak $(0, 5)$

Misal $y = 0$

$0 = x^2 - 6x + 5$

$(x - 5)(x - 1) = 0$

$x = 5 \text{ V } x = 1$

$(5, 0) \quad (1, 0)$

$0 = -x^2 + 5$

$x^2 - 5 = 0$

$(x - \sqrt{5})(x + \sqrt{5}) = 0$

$x = \sqrt{5} \text{ V } x = -\sqrt{5}$

$(\sqrt{5}, 0) \quad (-\sqrt{5}, 0)$

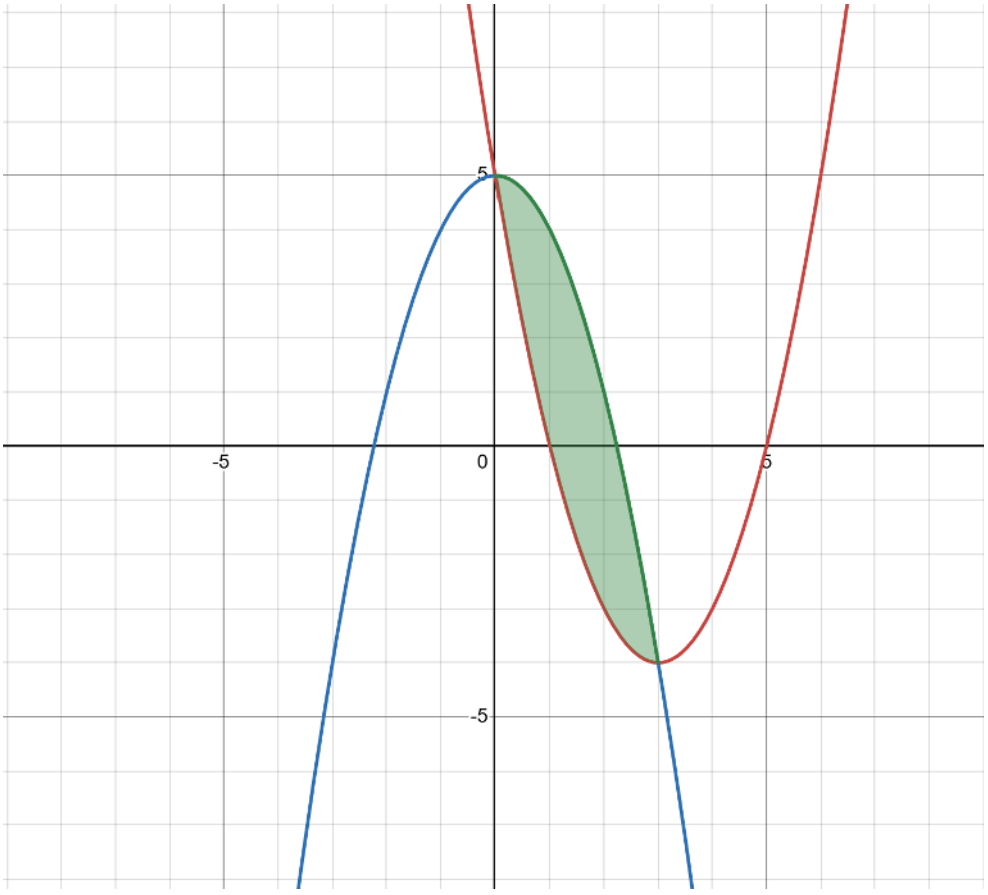
Cari titik potong

$-x^2 + 5 = x^2 - 6x + 5$

$2x^2 - 6x = 0$

$2x(x - 3) = 0$

$x = 0 \text{ V } x = 3$



Cari luas

$$L = \int_0^3 [-x^2 + 5 - (x^2 - 6x + 5)]\,dx$$

$$L = \int_0^3 [-x^2 + 5 - x^2 + 6x - 5]\,dx$$

$$L = \int_0^3 [-2x^2 + 6x]\,dx$$

$$L = [\tfrac{2}{3}x^3 + 3x^2]_0^3$$

$$L = -\tfrac{2}{3}(3^3 - 0^3) + 3(3^3 - 0^3)$$

$$L = -2(9) + 3(9)$$

$$L = 9\ satuan\ luas$$

C. 2 SOAL SURPLUS PRODUSEN DAN SURPLUS KONSUMEN

1. *Diketahui fungsi permintaan $P = 49 - Q^2$ dan fungsi penawaran $P = 2Q + 34$. Hitung besarnya surplus konsumen dan surplus produsen!*

JAWABAN =

Diketahui: Fungsi permintaan: $P_d = 49 - Q^2$

Fungsi penawaran: $P_s = 2Q + 34$

Ditanya: Surplus konsumen (CS) dan surplus produsen (PS)?

Jawab:

- Mencari jumlah keseimbangan (Qe)

  $P_d = P_s$

  $49 - Q^2 = 2Q + 34$

  $Q^2 + 2Q - 15 = 0$

  $(Q + 5)(Q - 3) = 0$

  $Q = -5\ atau\ Q = 3$

  (TM)

  $Q_e = 3$

- Mencari harga keseimbangan (Pe)

  $P = 2(3) + 34$

  $P = 6 + 34$

  $P_e = 40$

- Menghitung surplus konsumen (CS)

Rumus: $CS = \int_0^{Q_e} (P_d - P_e) dQ$

$$CS = \int_0^3 (49 - Q^2 - 40) dQ$$

$$CS = \int_0^3 (9 - Q^2) dQ$$

$$CS = \left[9Q - \frac{Q^3}{3} \right]_0^3$$

$$= 9(3) - \frac{3^3}{3}$$

$$= 27 - 9 = 18$$

CS = 18

- Menghitung surplus produsen (PS)

$$PS = \int_0^{Q_e} (P_e - P_s) dQ$$

$$PS = \int_0^3 (40 - 2Q - 34) dQ$$

$$PS = \int_0^3 (6 - 2Q) dQ$$

$$PS = \left[6Q - Q^2 \right]_0^3$$

$$= 6(3) - 3^2$$

$$= 18 - 9 = 9$$

PS = 9

Jadi besar surplus konsumen adalah 18, dan suplus produsen adalah 9

2. *The demand function for a certain commodity is $P = 240 - 8Q - 2Q^2$, and the supply function for these commodity is given by $P = 10Q + 4Q^2$, where P stands for the unit price in dollars and Q stands for the number of commodities. The producers' surplus on the equilibrium price is …*

JAWABAN =

Diketahui: Fungsi permintaan: $P_d = 240 - 8Q - 2Q^2$

Fungsi penawaran: $P_s = 10Q + 4Q^2$

Ditanya: Surplus produsen (PS)?

Jawab:

- Mencari harga dan jumlah keseimbangan

$$P_d = P_s$$
$$240 - 8Q - 2Q^2 = 10Q + 4Q^2$$
$$240 - 18Q - 6Q^2 = 0$$
$$6Q^2 + 18Q - 240 = 0$$
$$Q^2 + 3Q - 40 = 0$$
$$(Q + 8)(Q - 5) = 0$$
$$Q = -8 \; atau \; Q = 5$$

(TM)

$\underline{Q_e = 5}$

- Mencari harga keseimbangan (Pe)

$P = 10(5) + 4(5)^2$

$P = 50 + 100$

$P_e = 150$

- Menghitung surplus produsen (PS)

Rumus: $PS = \int_0^{Q_e} (P_e - P_s) dQ$

$$PS = \int_0^5 (150 - 10Q - 4Q^2) dQ$$

$$PS = \left[150Q - 5Q^2 - \frac{4}{3}Q^3\right]_0^5$$

$$= 150(5) - 5(5)^2 - \frac{4}{3}(5)^3 \backslash$$

$$= 750 - 125 - \frac{500}{3}$$

$$= \frac{2250-375-500}{3} = \frac{1375}{3} = 458{,}33$$

PS = $458,33

So, the producers' surplus on the equilibrium price is $458,33